

PUZZLE

In a certain place, all the inhabitants are either Knights or Knaves. Knights always tell the truth and Knaves never tell the truth.

You meet two inhabitants, A and B. B says “Both of us are Knaves.” What, if anything, can you infer from this?

ATOMIC SENTENCES

Friday, 17 January

ADMINISTRATIVE

- Course website:

<http://joelvelasco.net/teaching/2310/>

- Assignment 1 is posted (due Fri, Jan 24th)

WHAT IS FOL?

- Actually, some languages are ‘first order’ languages, others aren’t. First order logic (FOL) could be thought of as the study of all first order languages.
 - But we will often talk as though FOL is a specific language
- All first order languages have certain structural features (like a common grammar)

SENTENCES IN FOL

- Sentences in FOL are either true or false in any particular world (model).
- An atomic sentence is a predicate associated with the appropriate number of terms.

CONSTANTS

- Constants are symbols that pick out objects in the universe.
- Each constant must name one (and only one) object.
- An object can be referred to by more than one constant, or by no constant at all.

PREDICATES

- Predicates are symbols used to ascribe properties or relations to objects in our universe.
- Each predicate takes a fixed number of objects; this is called its arity (unary, binary, ternary, etc.)
- All predicates in FOL express determinate properties and relations.

SYNTAX

- Our book uses the following syntax for an atomic sentence:
 - A predicate starts with a capital letter
 - Parentheses surround the terms which are separated by commas

SPECIAL EXCEPTION: IDENTITY

- Identical(a,b) would be an atomic sentence
- As an abbreviation, we use $a=b$ for this sentence
- For technical reasons, we want identity to be a logical relation (always means the same thing in every world)

FUNCTION SYMBOLS

- A constant is a special type of term. A function applied to a term (like a constant) yields another term. A function applied to a term refers to exactly one object.
- Like predicates, each function symbol takes a fixed number of objects (has a fixed arity).
- Unlike predicates, when you combine a function with a term, you do not get a sentence but another term.

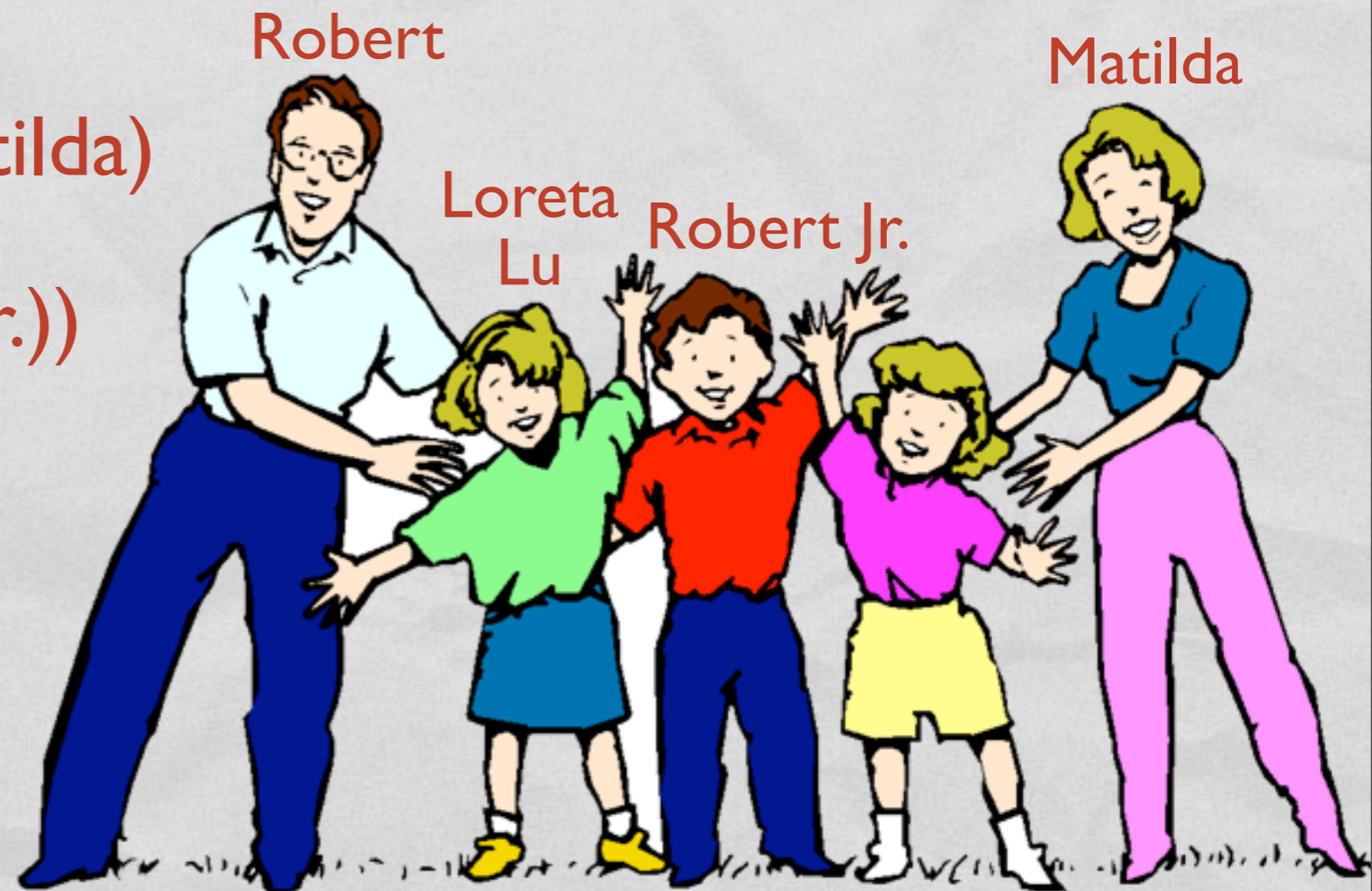
ATOMIC SENTENCES

Male(Loreta):

Lu = Loreta

Shorter(Robert Jr., Matilda)

Male(mother(Robert Jr.))



ATOMIC VS. SIMPLE

Atomic sentences are sometimes simple - like $P(a)$ but sometimes not like:

Between(mother(Robert), tallerof(Robert, Lu), father(mother(Anita)))

Or:

The least prime number that is greater than 25 equals the square of the greatest common factor of 102 and 120 minus 7.

Complex sentences include:

Bob and Jane are tall

$5 \neq 7$

FIRST-ORDER LANGUAGES

- Different constants, functions, and predicates; so different sentences can be formed
- Language 1: Robert, Lu, Tall, motherof, etc.
- Language 2: Arithmetic: 1, 2, Prime, Even, +, *, etc.
- Language 3: Blocks Language in Tarski's World

BLOCKS LANGUAGE

- The blocks language is used in Tarski's world.
 - The constants are a,b,c,d,e,f
 - Predicates include Cube, Small, LeftOf, SameCol, and others
- So atomic sentences are things like Cube(a), Small(b), LeftOf(a,c), SameCol(b,b)

TARSKI'S WORLD

Blocks Language

Constants

\wedge	\vee	\neg	\rightarrow	\leftrightarrow	\perp
a	b	c	d	e	f
\forall	\exists	=	\neq	()
x	y	z	u	v	w


Blocks					Pets	Set	Arith
Tet	Small	LeftOf	SameCol	Smaller			
Cube	Medium	RightOf	SameRow	Larger			
Dodec	Large	FrontOf	Between				
SameShape	SameSize	BackOf	Adjoins				

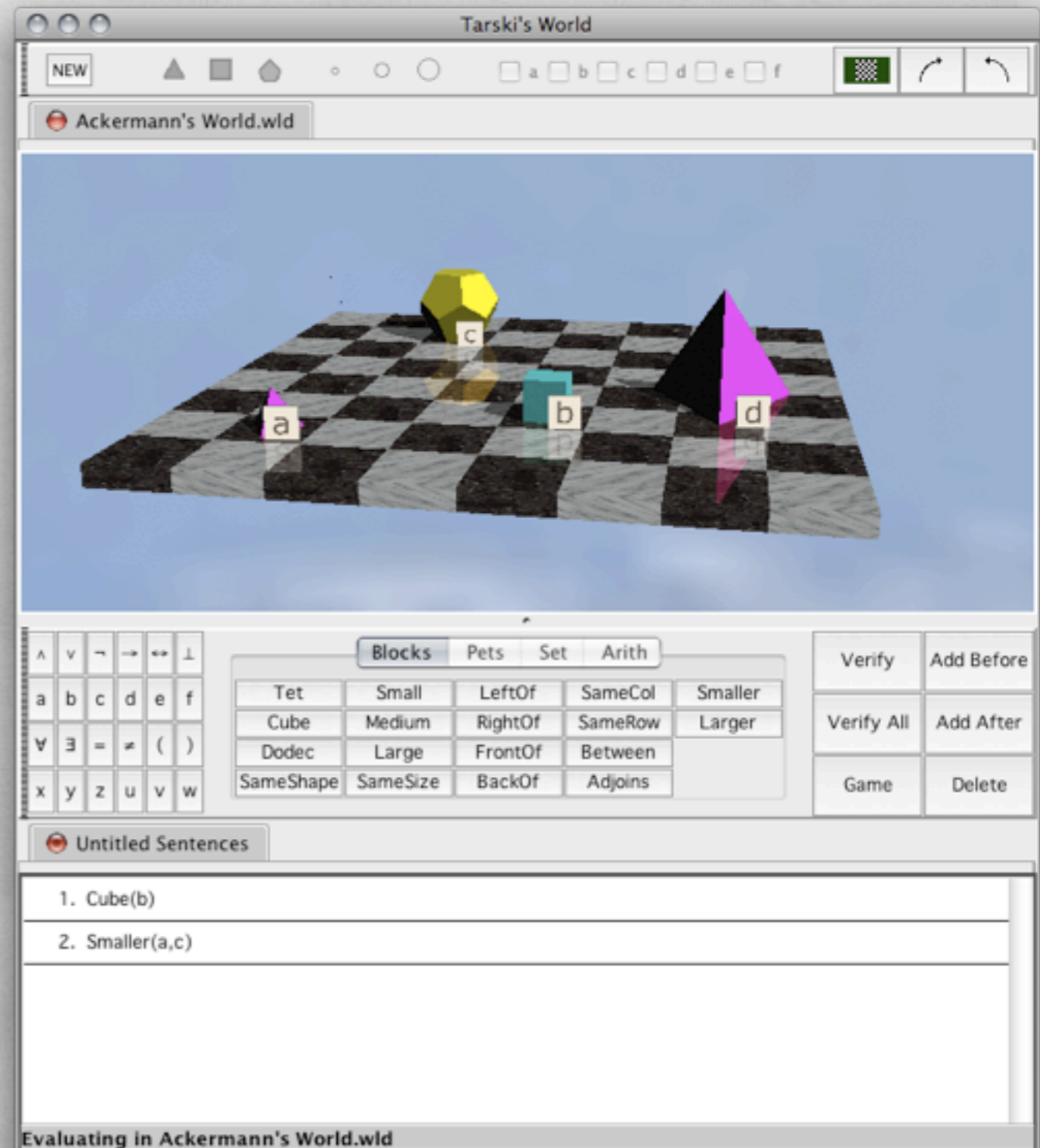
Predicates

TARSKI'S WORLD

Using Tarski's World

World 

Atomic Sentences 



WHERE WE ARE

- You should now be able to complete the first part of the homework on atomic sentences (all from Chap 1 in the book).
- A small portion of the homework is on validity which is the beginning of chapter 2 (W lecture).