PROOF THAT LOGIC IS AWESOME

Call the following sentence P: "If this conditional is true, then Logic is Awesome". I will now prove that P is in fact true. P is a conditional. I will assume its antecedent and then show that its consequent would follow. So we assume that "this conditional is true". "this conditional" refers to sentence P so I now have by assumption that P is true. But since P is true and we also have its antecedent, then its consequent follows by modus ponens so given my assumption of "this conditional is true" then Logic is Awesome so by conditional proof (→Intro), I have now proved P. But then the antecedent "this conditional is true" is in fact true since I have just proved that P is true. Therefore by modus ponens, it follows that:

Logic is Awesome.

Summary of the Course

Monday, 5 May

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- In one view, Logic is the study of the logical consequence relation. (Or maybe better, deductive logic)

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- One way to tell the story of the class is to look at a series of ways of getting better and better at formalizing logical consequence and finally, examining fundamental facts about the scope and limits of formal systems themselves.
- Step 1: Develop a formal language for describing the propositions and arguments of interest. We have FOL which is the foundation of all natural and artificial languages that we know of.

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- Some sentences are atomic - they don't have any smaller sentences as parts. These have a subject-predicate form. (?)
- Complex sentences are formed from atomic sentences by recursive rules. These rules involve connectives and logical operators. Many connectives are truth-functional, many operators are quantificational in nature.

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- There are many different systems. It doesn't matter which you use. What matters is that it perfectly matches the truth-tables (soundness and completeness).

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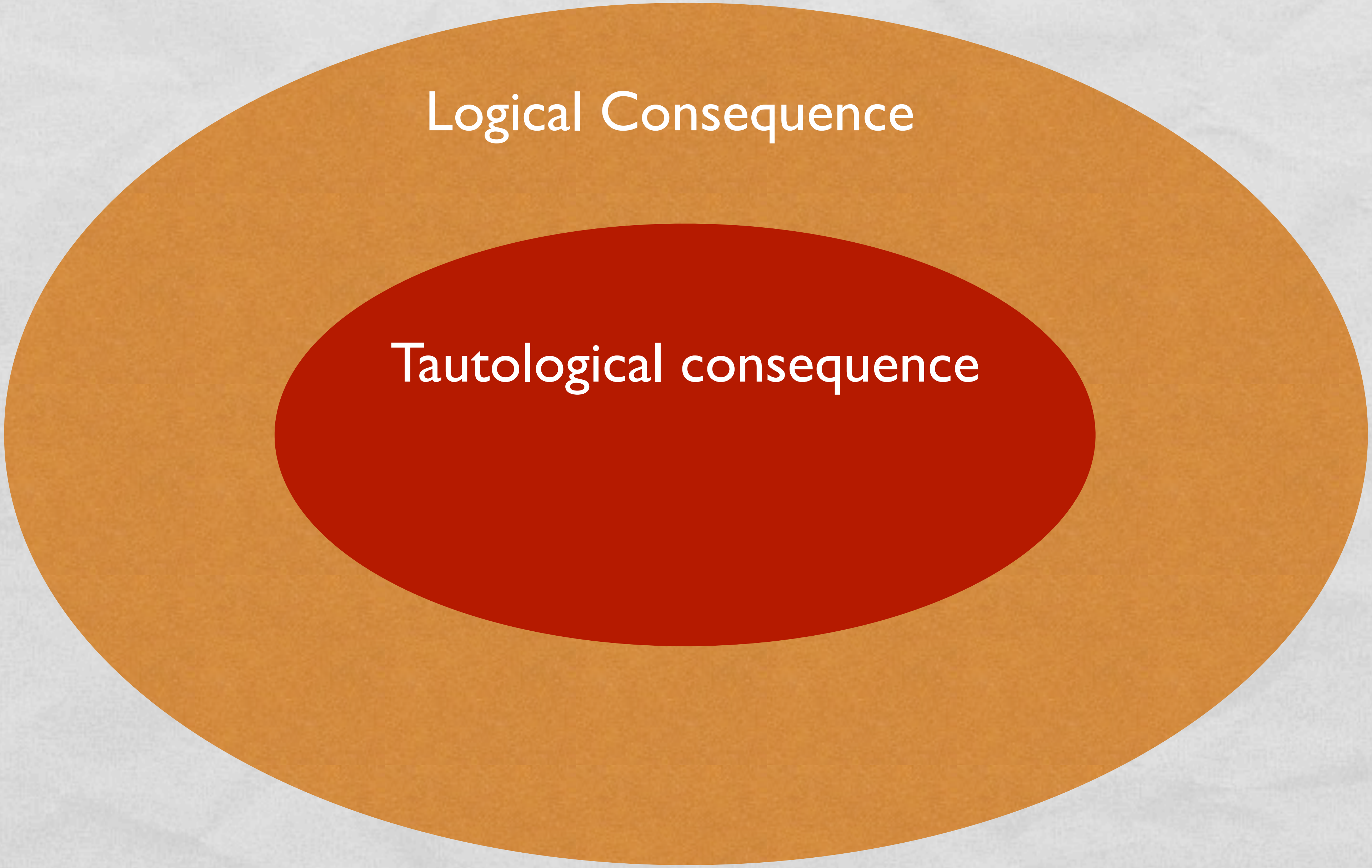

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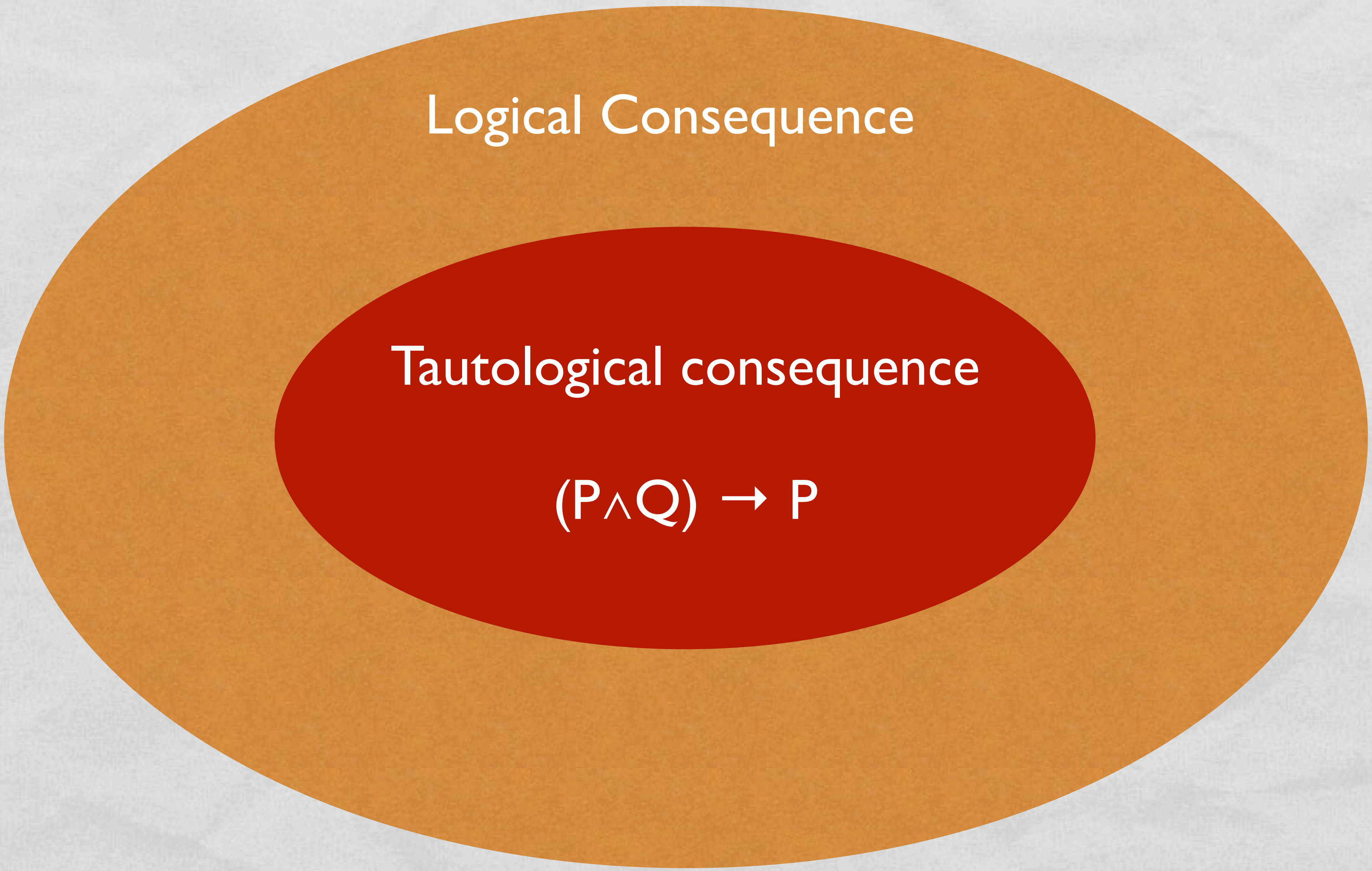

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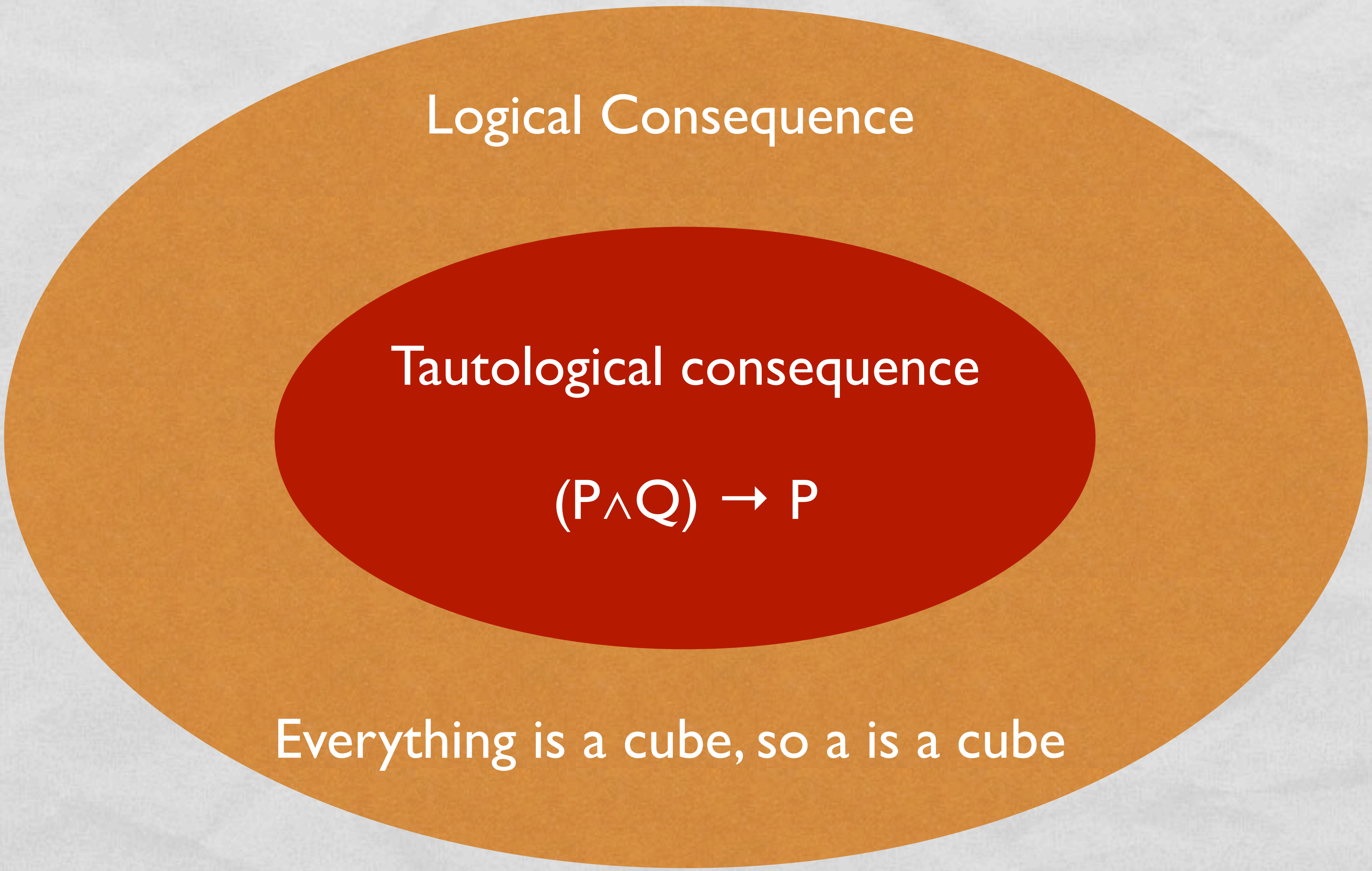

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- The next step is to look at the quantificational structure. Many types of quantification are equivalent to some combination of 'all' and 'some'.
- We are still concerned with consistency and 'all possible circumstances'. But now, a possible circumstance is there could be any number of things and any particular predicate could be true of any number of them, relation true of any pair, etc.

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- Sometimes 'logical consequence' is just defined this way as FO consequence. (Similarly, logical consistency, possibility, etc.) Here the idea is that the quantifiers, connectives, and identity are logical symbols, the predicates aren't. Things can be true in our world because of facts about our world, or they might be logically true because of FOL.

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- But we could also say that validity is provability in some proof system - like $\mathcal{F}$. Historically, we had to agree what counted as FO valid and we first had a proof system which seemed to capture this. Now we have model theory with a set-theoretic description of what a model is that matches (sound + complete) the proof rules.

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- For example, sentential logic is decidable. There is a single algorithm (eg truth tables) that will correctly decide the truth of any question like 'is P a tautology' in finite time.
- However, with a language with at least one two (or more) place predicate, theoremhood (and consistency, entailment, etc.) are undecidable. There is no algorithm which will answer all questions of the form "Is P an FO validity?"

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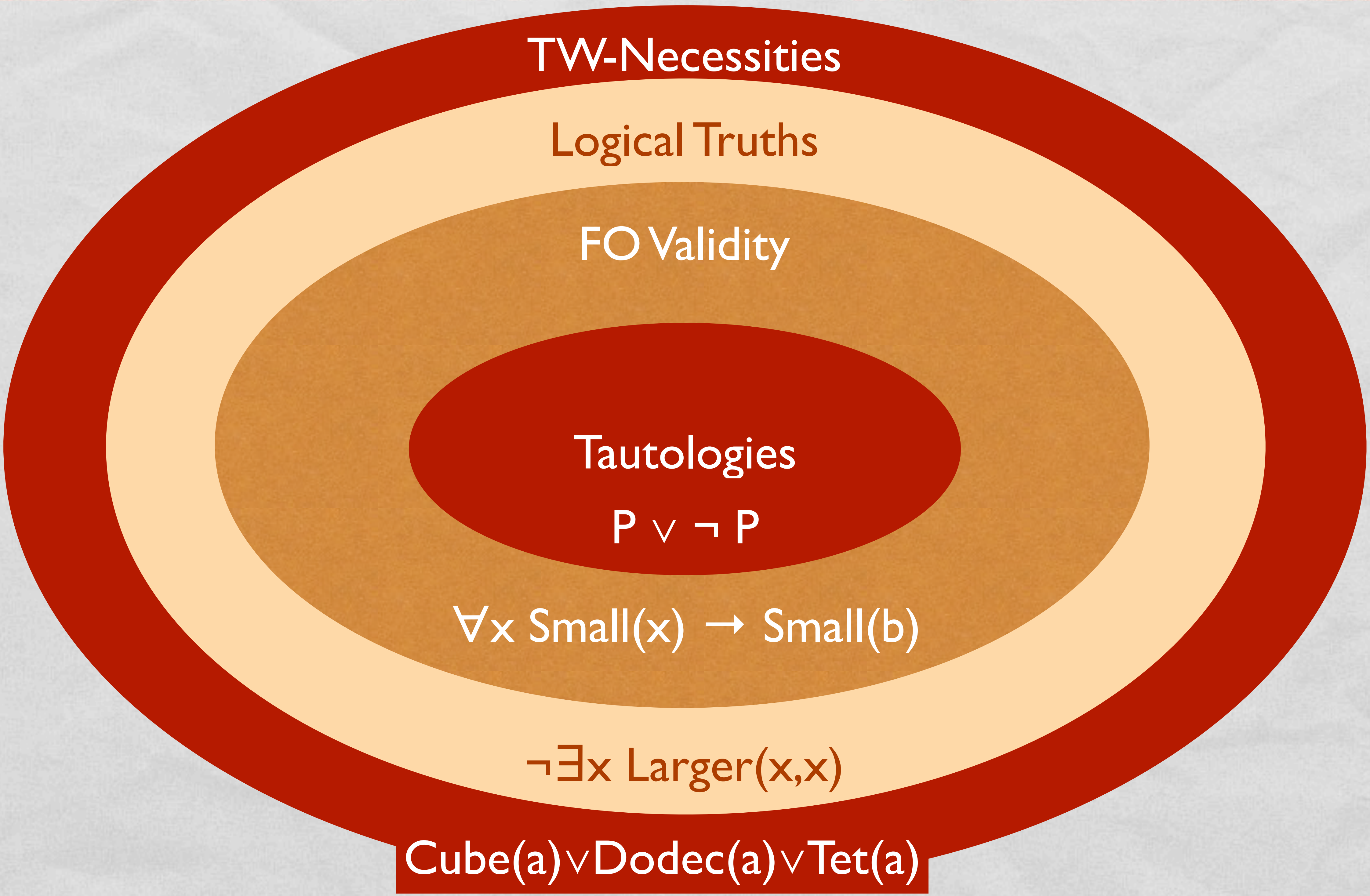
- FO validity captures consequence in virtue of the logical terms. But some arguments are valid in virtue partially of the meaning of the predicates. Our book thinks of these as logical truths. You don't have to, but you should have something to say about them.

First-Order Validity and Consequence



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- One description of this is that it is an 'analytic truth'. It is sometimes thought that FOL can capture the (logical?) truth of this sentence by adding meaning postulates, definitions, axioms, or whatever to transform this into an FO-validity.

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- Valid argument: More than half of the Ps are Qs. So there are more things that are $P \wedge Q$ than are $P \wedge \neg Q$.

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You can't say that 'logic doesn't apply here'. Ever.