

# PUZZLE

In a certain place, all the inhabitants are either Knights or Knaves. Knights always tell the truth and Knaves never tell the truth.

You meet two inhabitants, A and B. A says “Exactly one of us is a knight.” B says “A is a knight.” What, if anything, can you infer from this?



# CONDITIONALS

Wednesday, 29 January



# *Philosophy Spring 2014 Speaker Series*

*“Are Psychopaths Responsible?”*



*Walter Sinnott-Armstrong,  
Chauncey Stillman Professor of Practical Ethics  
Duke University  
Thursday, January 30, 2014 at 7:00 PM  
MCOM 353*

Abstract: Psychopaths are less than 1% of the population but commit over 30% of the violent crime in our country. They are widely misunderstood, but new studies (including some brain scans) have taught us a lot about what makes them tick. This new information points towards innovative psychiatric treatments and raises question about whether they should be held legally responsible.

*This program was made possible in part by grants from Humanities Texas, the state affiliate of the National Endowment for the Humanities as well as from the Ethics Center at Texas Tech University.*



# CLASS ANNOUNCEMENTS

- No office hours Thursday
- Class website: <http://joelvelasco.net/teaching/2310>
- **MUST** have a new copy of the book/software
  - You are really paying for your own software license and registration ID



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  - $(\text{Small}(a) \wedge \text{Small}(b)) \vee (\text{Large}(a) \wedge \text{Large}(b))$



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- The first is called the material conditional, designated with the symbol  $\rightarrow$ .
- If A and B are sentences, then  $A \rightarrow B$  is a sentence.



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- Bill is tall if Alice is:  $A \rightarrow B$
- If Bill and Alice are both tall, then neither Charlie nor David are:  $(B \wedge A) \rightarrow \neg(C \vee D)$



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  - Think “Alice will go (IF NOT) Tom



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A	B	$A \rightarrow B$
TRUE	TRUE	TRUE
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FALSE	TRUE	TRUE
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In FOL, if the consequent is true, then the conditional is always true.
- $A \rightarrow B$  just means either  $A$  is false or  $B$  is true



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