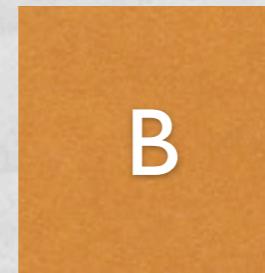


You are given four cards with a number on one side and a letter on the other. You can only see one side of each card.



Which card(s) do you have to turn over in order to fully test the following rule:

If there is a vowel on one side of the card,
then there is an even number on the other side.

You are given four cards with a drink on one side and an age on the other. You can only see one side of each card.

beer

pepsi

16

30

Which card(s) do you have to turn over in order to fully test the following rule:

If you are drinking alcohol, then you must be over 21.

PROOFS WITH CONDITIONALS

Monday, 20 September

RULES FOR CONDITIONALS

- \rightarrow Elimination: from $P \rightarrow Q$ and P , we can infer Q .

$$\frac{\begin{array}{l} 1. P \rightarrow Q \\ 2. P \end{array}}{3. Q} \rightarrow \text{Elim: I,2}$$

- \leftrightarrow Elimination: from $P \leftrightarrow Q$ and P/Q , we can infer Q/P .

$$\frac{\begin{array}{l} 1. P \leftrightarrow Q \\ 2. Q \end{array}}{3. P} \leftrightarrow \text{Elim: I,2}$$

RULES FOR CONDITIONALS

1. $P \rightarrow Q$

2. P

3. Q

→ Elim: 1,2

1. $P \rightarrow Q$

2. $\neg P$

3. $\neg Q$

INVALID

1. $P \rightarrow Q$

2. Q

3. P

INVALID

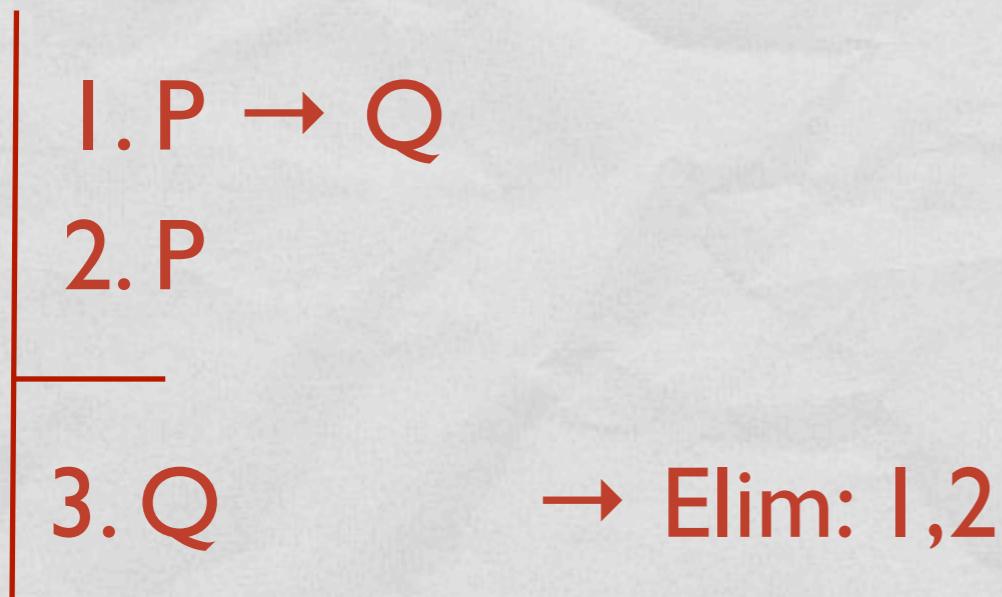
1. $P \rightarrow Q$

2. $\neg Q$

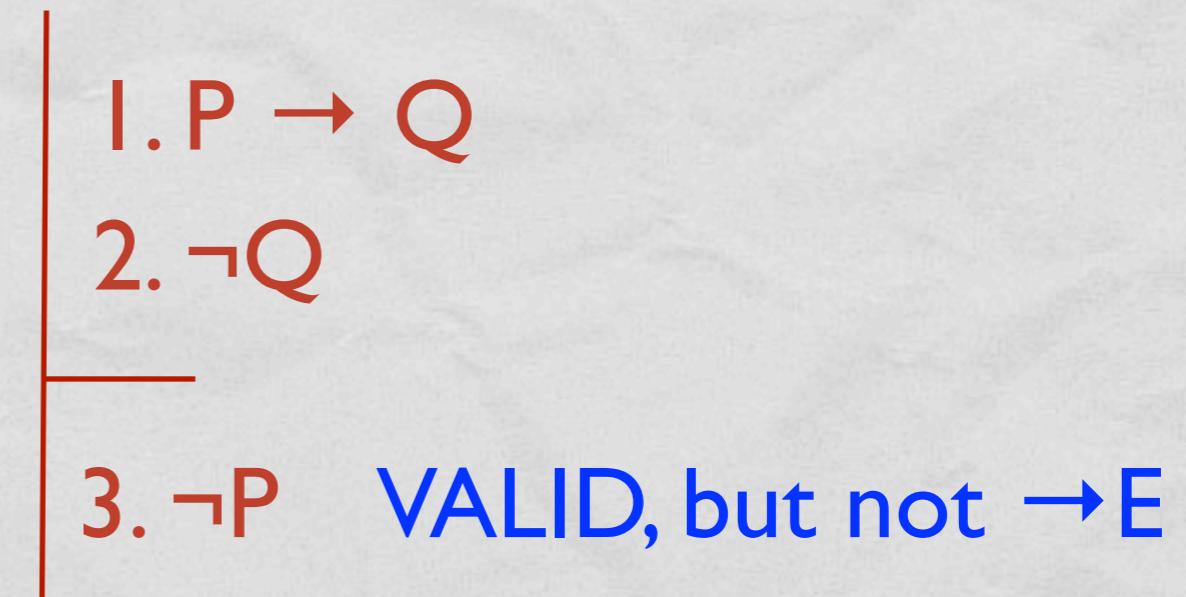
3. $\neg P$

VALID, but not $\rightarrow E$

RULES FOR CONDITIONALS



Modus Ponens



Modus Tollens

EXAMPLE

Example:

$$P \vee Q$$

$$P \rightarrow R$$

$$Q \leftrightarrow \neg S$$

$$S \vee R$$

$$R$$

$$1. P \vee Q$$

$$2. P \rightarrow R$$

$$3. Q \leftrightarrow \neg S$$

$$4. S \vee R$$

$$5. P$$

for \vee Elim

$$R$$

$$Q$$

for \vee Elim

$$R$$

$$R$$

\vee Elim 1, 5...

EXAMPLE

Example:

$$P \vee Q$$

$$P \rightarrow R$$

$$Q \leftrightarrow \neg S$$

$$S \vee R$$

$$\underline{R}$$

$$1. P \vee Q$$

$$2. P \rightarrow R$$

$$3. Q \leftrightarrow \neg S$$

$$4. S \vee R$$

$$5. P$$

for \vee Elim

$$6. R$$

\rightarrow Elim 2,5

$$7. Q$$

for \vee Elim

$$8. \neg S$$

\leftrightarrow Elim 3,7

$$R$$

by disjunctive syllogism 4,8

$$R$$

\vee Elim 1, 5...

RULES USING CONTRADICTIONS

Example: Disjunctive Syllogism

$$\begin{array}{c} P \vee Q \\ \neg P \\ \hline Q \end{array}$$

$$\begin{array}{c} 1. P \vee Q \\ 2. \neg P \\ \hline 3. P \\ \hline 4. \perp \\ 5. Q \\ \hline 6. Q \\ \hline 7. Q \end{array}$$

for \vee Elim
 \perp Intro 2,3
 \perp Elim 4
for \vee Elim
 \vee Elim 1,3-5,6-6

RULES USING CONTRADICTIONS

Example: Disjunctive Syllogism

$$\begin{array}{c} P \vee Q \\ \neg P \\ \hline Q \end{array}$$

$$\begin{array}{c} 1. S \vee R \\ 2. \neg S \\ \hline 3. S \\ \hline 4. \perp \\ 5. R \\ \hline 6. R \\ \hline 7. R \end{array}$$

for \vee Elim
 \perp Intro 2,3
 \perp Elim 4
for \vee Elim
 \vee Elim 1,3-5,6-6

EXAMPLE

$$1. P \vee Q$$

$$2. P \rightarrow R$$

$$3. Q \leftrightarrow \neg S$$

$$4. S \vee R$$

$$5. P$$

$$6. R$$

$$7. Q$$

$$8. \neg S$$

$$R$$

R by disjunctive syllogism 4,8

\vee Elim 1, 5-...

Insert DS proof here

for \vee Elim

\rightarrow Elim 2,5

for \vee Elim

\leftrightarrow Elim 3,7

$$3. S$$

$$4. \perp$$

$$5. R$$

$$6. R$$

$$7. R$$

for \vee Elim

\perp Intro 2,3

\perp Elim 4

for \vee Elim

\vee Elim 1,3-5,6-6

EXAMPLE

I. $P \vee Q$

2. $P \rightarrow R$

3. $Q \leftrightarrow \neg S$

4. $S \vee R$

5. P

for \vee Elim

6. R

\rightarrow Elim 2,5

7. Q

for \vee Elim

8. $\neg S$

\leftrightarrow Elim 3,7

Insert DS proof here

9. S

for \vee Elim

10. \perp

\perp Intro 8,9

11. R

\perp Elim 10

12. R

for \vee Elim

13. R

\vee Elim 4,9-11,12-13

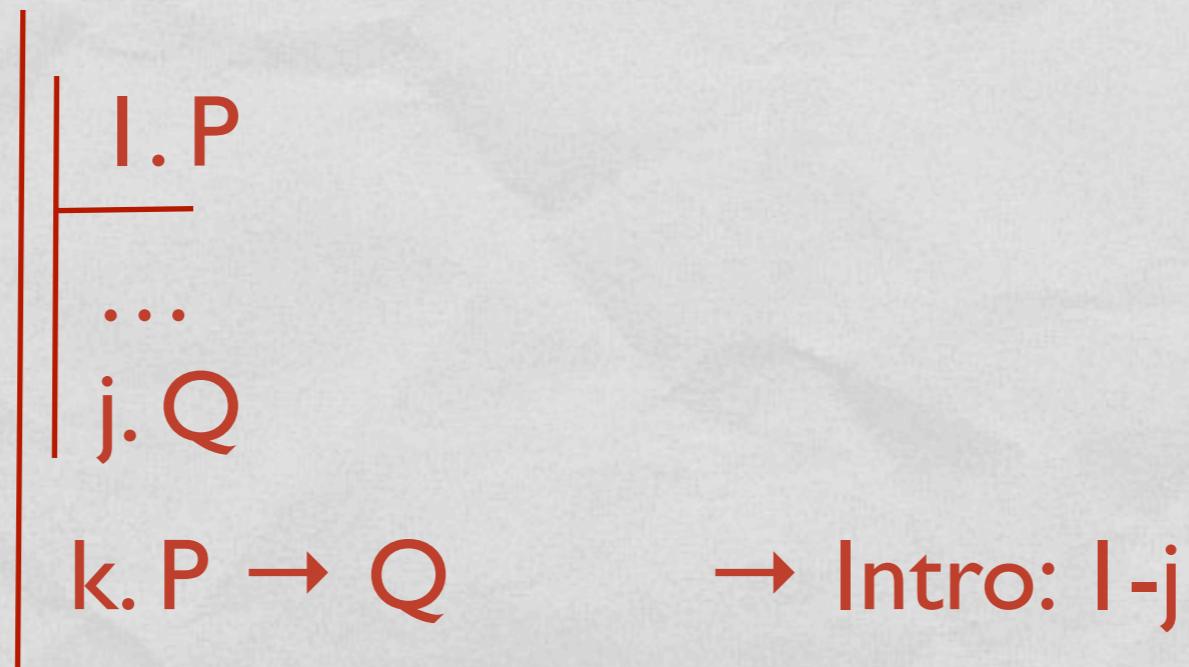
13. R \vee Elim 4,9-11,12-13

14. R \vee Elim 1,5-6,7-13

FORMAL PROOF RULES

- → Introduction

From a proof from P to Q , we can infer $P \rightarrow Q$.



This rule is often known as Conditional Proof

CHAIN ARGUMENT

Example:

$$\begin{array}{l} P \rightarrow Q \\ Q \rightarrow R \\ \hline P \rightarrow R \end{array}$$

$$\begin{array}{l} 1. P \rightarrow Q \\ 2. Q \rightarrow R \\ \hline 3. P \\ \hline 4. Q \\ 5. R \\ \hline 6. P \rightarrow R \end{array}$$

for \rightarrow Intro
 \rightarrow Elim 1,3
 \rightarrow Elim 2,4
 \rightarrow Intro 3-5

TRANSITIVITY OF \rightarrow

Example:

$$\frac{P \rightarrow Q}{(Q \rightarrow R) \rightarrow (P \rightarrow R)}$$

$$\begin{array}{ll} \text{I. } P \rightarrow Q & \\ \hline & 2. \quad Q \rightarrow R \quad \text{for } \rightarrow \text{Intro} \\ & | \\ & 3. \quad P \quad \text{for } \rightarrow \text{Intro} \\ & | \\ & 4. \quad Q \\ & | \\ & R \\ & | \\ & P \rightarrow R \quad \rightarrow \text{Elim I,3} \\ & | \\ & (Q \rightarrow R) \rightarrow (P \rightarrow R) \quad \rightarrow \text{Intro} \end{array}$$

TRANSITIVITY OF \rightarrow

Example:

$$\frac{P \rightarrow Q}{(Q \rightarrow R) \rightarrow (P \rightarrow R)}$$

I. $P \rightarrow Q$	
2. $Q \rightarrow R$	for \rightarrow Intro
3. P	for \rightarrow Intro
4. Q	\rightarrow Elim 1,3
5. R	\rightarrow Elim 2,4
6. $P \rightarrow R$	\rightarrow Intro 3-5
7. $(Q \rightarrow R) \rightarrow (P \rightarrow R)$	\rightarrow Intro 2-6

NOTICE THE STRUCTURE

1. $P \rightarrow Q$

2. $Q \rightarrow R$

3. P

for \rightarrow Intro

4. Q

\rightarrow Elim 1,3

5. R

\rightarrow Elim 2,4

6. $P \rightarrow R$

\rightarrow Intro 3-5

1. $P \rightarrow Q$

2. $Q \rightarrow R$

for \rightarrow Intro

3. P

for \rightarrow Intro

4. Q

\rightarrow Elim 1,3

5. R

\rightarrow Elim 2,4

6. $P \rightarrow R$

\rightarrow Intro 3-5

7. $(Q \rightarrow R) \rightarrow (P \rightarrow R)$

\rightarrow Intro 2-6

SUBPROOFS AND PROOFS

$$\begin{array}{c} P \rightarrow Q \\ Q \rightarrow R \\ \hline P \rightarrow R \end{array}$$

$$\begin{array}{c} P \rightarrow Q \\ \hline (Q \rightarrow R) \rightarrow (P \rightarrow R) \end{array}$$

$$\begin{array}{c} P \rightarrow Q \\ Q \rightarrow R \\ P \\ \hline R \end{array}$$

$$\begin{array}{c} \hline (P \rightarrow Q) \rightarrow [(Q \rightarrow R) \rightarrow (P \rightarrow R)] \end{array}$$

MODUS TOLLENS

Example:

$$\begin{array}{c} P \rightarrow Q \\ \neg Q \\ \hline \neg P \end{array}$$

$$\begin{array}{ll} 1. P \rightarrow Q & \\ 2. \neg Q & \\ \hline & 3. P \quad \text{for } \neg \text{Intro} \\ \hline & 4. Q \quad \rightarrow \text{Elim 1,3} \\ & 5. \perp \quad \perp \text{Intro 2,4} \\ \hline & 6. \neg P \quad \neg \text{Intro 3-5} \end{array}$$

CONTRAPosition

Example:

$$\begin{array}{c} P \rightarrow Q \\ \hline \neg Q \rightarrow \neg P \end{array}$$

$$\begin{array}{l} 1. P \rightarrow Q \\ \hline \begin{array}{l} 2. \neg Q \\ \hline \begin{array}{l} 3. P \\ \hline \begin{array}{l} 4. Q \\ \hline \begin{array}{l} 5. \perp \\ \hline \begin{array}{l} 6. \neg P \\ \hline \begin{array}{l} 7. \neg Q \rightarrow \neg P \end{array} \end{array} \end{array} \end{array} \end{array} \end{array}$$

for \rightarrow Intro

for \neg Intro

\rightarrow Elim 1,3

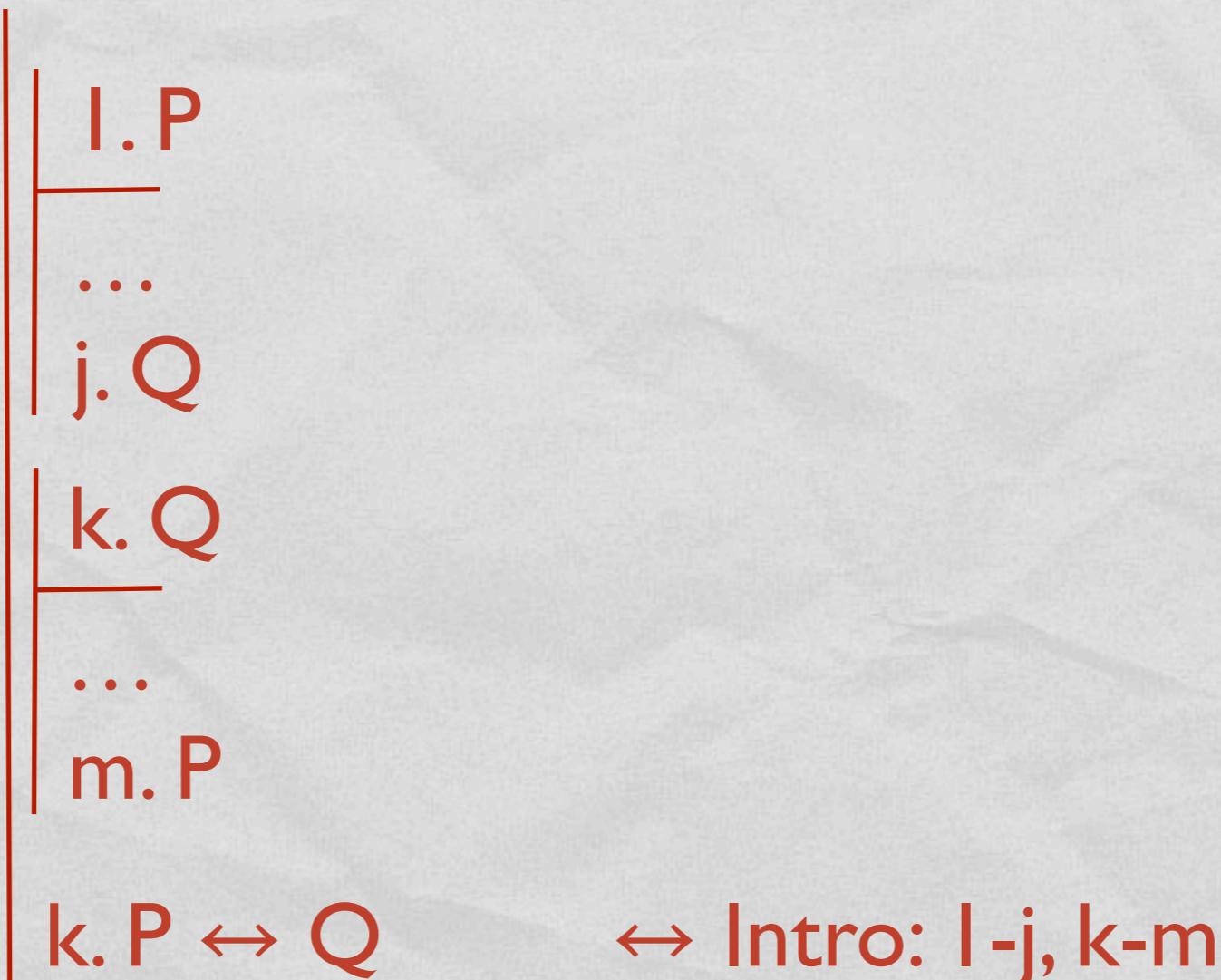
\perp Intro 2,4

\neg Intro 3-5

\rightarrow Intro 2-6

FORMAL PROOF RULES

- \leftrightarrow Introduction: from a proof from P to Q and a proof from Q to P , we can infer $P \leftrightarrow Q$.



BICONDITIONALS

Example:

$$\begin{array}{c} P \leftrightarrow Q \\ | \\ Q \leftrightarrow R \\ | \\ P \leftrightarrow R \end{array}$$

1. $P \leftrightarrow Q$	
2. $Q \leftrightarrow R$	
3. P	for \leftrightarrow Intro → Elim 1,3 → Elim 2,4
4. Q	
5. R	
6. R	for \leftrightarrow Intro → Elim 1,3 → Elim 2,4
7. Q	
8. P	
$P \leftrightarrow R$	\leftrightarrow Intro 3-5, 6-8