Phil 5312
Fall 2024

Assignment 3:
Finish reading MacFarlane chapter 1. Then do the exercises below. Answers should be uploaded into Blackboard by Wednesday, Sept 25th. This can be typed up or parts can be written and then you can upload a picture or scan. If you don't know how to do the problems, you should talk to me and/or your fellow students. Collaboration is totally fine (and encouraged). But the final work product should be your own work.

We are aiming for mastery of chapter 1. So if you make a good faith effort and still can't solve a problem or make a mistake, I will allow you to redo some problems.

Part 1. Soundness and Completeness

Give an informal proof that for any propositional sentence *p* exactly one of the following three conditions holds:

(a) There is a formal proof (from no premises) of *p* in the system $\mathcal{F}_M$ (MacFarlane's proof system).

(b) There is a formal proof (from no premises) of ¬*p* in the system $\mathcal{F}_M$.

(c) The truth table for *p* contains at least one line which makes *p* true, and at least one line which makes it false.

For this problem, you may assume without argument that both the Soundness Theorem and Completeness Theorem hold for $\mathcal{F}_M$. But if you use either of these theorems in your answer, be sure to indicate clearly which theorem you are using and exactly where you are using it in your proof.

Part 2: Assume that it is possible to construct a proof in $\mathcal{F}_M$ (MacFarlane's proof system) from the premises P_1, P_2, P_3 to the conclusion Conc. Which of the following MUST be true? (The correct answer may be any number of these).

1) Conc is a logical consequence of $\{P_1, P_2, P_3\}$
2) ¬Conc is not a logical consequence of $\{P_1, P_2, P_3\}$
3) $\{P_1, P_2, P_3\}$ is a consistent set
4) $\{P_1, P_2, P_3\}$ is an inconsistent set
5) $\{P_1, P_2, P_3, \text{Conc}\}$ is an inconsistent set
6) $\{P_1, P_2, P_3, \neg\text{Conc}\}$ is an inconsistent set
7) $\{P_2, P_3, \neg\text{Conc}\}$ is an inconsistent set
8) $\{P_2, P_3, \neg\text{Conc}\}$ is a consistent set
9) $\{\neg P_1, P_2, P_3, \text{Conc}\}$ is an inconsistent set
10) $\{\neg P_1, \neg P_2, \neg P_3, \text{Conc}\}$ is a consistent set

11) $\neg P_1$ is a logical consequence of $\{P_2, P_3, Conc\}$
12) $\neg P_1$ is a logical consequence of $\{P_2, P_3, \neg Conc\}$
13) $\neg P_3$ is provable in $\mathcal{F}_M$ from $\{P_1, P_2, \neg Conc\}$
14) P_3 is provable in $\mathcal{F}_M$ from $\{P_1, P_2, Conc\}$
15) $P_1 \supset Conc$ is provable in $\mathcal{F}_M$ from $\{P_2, P_3\}$
16) $P_1 \equiv Conc$ is provable in $\mathcal{F}_M$ from $\{P_2, P_3\}$
17) $\neg Conc \supset \neg P_3$ is provable in $\mathcal{F}_M$ from $\{P_1, P_2\}$
18) $(P_1 \wedge P_2 \wedge P_3) \supset Conc$ is provable in $\mathcal{F}_M$ from $\{\ \}$
19) $(\neg P_1 \wedge \neg P_2 \wedge \neg P_3) \supset \neg Conc$ is not provable in $\mathcal{F}_M$ from $\{\ \}$
20) $P_1 \supset (P_2 \supset (P_3 \supset Conc))$ is a logical truth
21) $\neg Conc \supset (\neg P_1 \wedge \neg P_2 \wedge \neg P_3)$ is a logical truth

Part 3. Which of the above MUST be false? (Hint: the answer is not just everything that wasn't correct in Part 2).

Part 4. Do exercise 1.6 on page 30.

Part 5: Diagrams
Read the diagrams supplement on Blackboard on evaluating quantifier sentences. The sample diagram problems might also be helpful.

Now determine which of these sentences are true on which of these diagrams. For example, a 4x6 grid of 24 true/false answers is one way to answer this. It might help to think about teaching attending meetings.

1. $\exists x(Mx \wedge \forall y(Ty \supset \neg Ayx))$
2. $\forall x(Mx \supset \exists y \exists z(Ty \wedge Tz \wedge Ayx \wedge \neg Azx))$
3. $\forall x(Tx \supset \exists y \exists z(My \wedge Mz \wedge y \neq z \wedge Axy \wedge Axz))$
4. $\forall x \forall y((Mx \wedge My \wedge x \neq y) \supset \exists z(Tz \wedge Azx \wedge Azy))$
5. $\exists x \exists y(Tx \wedge Ty \wedge x \neq y \wedge \forall z(Mz \supset (Axz \equiv Ayz)))$
6. $\exists x(Mx \wedge \forall y \forall z((Ty \wedge Tz \wedge y \neq z) \supset (Ayx \vee Azx)))$

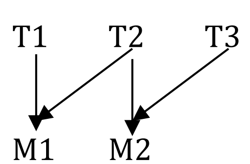
Diagram 1


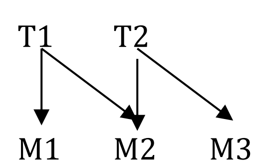
Diagram 2


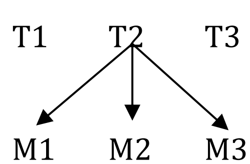
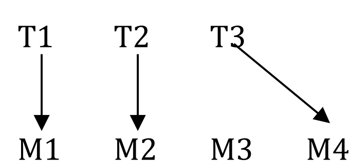
Diagram 3


Diagram 4


Part 6: Proofs and Countermodels

For each sequent, determine whether or not it is valid. If it is valid, give a proof. If it is not valid, produce a formal countermodel. For proofs, you may use any shortcut rules we have talked about. For example, anything valid in propositional logic, Quantifier negation exchange rules, and introducing a negated identity claim can all be done in one step. Multiple, sequential quantifier rules of the same type can also be done in one step (such as plugging in two constants for two consecutive universal quantifiers). A formal countermodel consists of a set for the domain and a set for each of the one-place predicates and a set of ordered pairs for the two place predicates.

1. $\exists x(Px \wedge \forall y(x \neq y \supset Rxy)) \vdash \forall x(\neg Px \supset \exists y(y \neq x \wedge Ryx))$
2. $\forall x(Px \vee Qx), \exists x \neg Px \wedge \exists y \neg Qy \vdash \forall x(Px \supset \neg Qx)$
3. $\exists x \forall y(x=y \supset Px), \forall x \forall y((Px \wedge Py) \supset x=y) \vdash \exists x(Px \wedge \neg \exists y(Py \wedge x \neq y))$
4. $\forall x Rxx, \exists x \exists y \exists z(Rxy \wedge Ryz \wedge \neg Rxz) \vdash \exists x \exists y \exists z(x \neq y \wedge x \neq z \wedge y \neq z)$
5. $\exists x \forall y\ x=y, \neg \forall x Px \vdash \forall x \neg Px$
6. $\forall x(Px \supset \exists y(Qy \wedge Rxy)), \forall x(Qx \supset \exists y(Py \wedge Rxy)) \vdash \forall x \forall y(Rxy \supset Ryx)$
7. $\forall x(\exists y Rxy \supset \exists y \exists z(y \neq z \wedge Rxy \wedge Rxz)), \exists x \forall y Rxy$
   $\vdash \forall x \forall y \forall z((Rxy \wedge Ryz) \to Rxz)$
8. $\forall x \exists y(Rxy \wedge \neg Ryx) \vdash \exists x \exists y \exists z(x \neq y \wedge y \neq z \wedge x \neq z)$