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INTERPRETING QUANTIFICATION

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Alternative readings of quantification are considered. The absence of an unequivocal translation into ordinary speech is noted. Some examples are cited which, in the opinion of the author, are a result of equivocal readings of quantification, or unnecessarily restrictive readings which obscure its primary function.

The logical operators of the propositional calculus have had a fairly clear-cut interpretation. An exception is material implication, which is seen to be inadequate as a formalization of the entailment relation. Other modifications of interpretation are noted, such as the restriction of conjunction and disjunction to commutative uses of 'and' and 'or', but these are straightforward. When we come to the functional calculus, there is disagreement as to how to read the operations of quantification. In addition, given some agreement about how to read the operations of quantification, there is considerable divergence as to what is entailed by such a reading. Recently, one of the fruitful interpretations of quantification seems to have been abandoned or at least submerged. I should like to restate the interpretation, and consider some of the controversy which it obviates.

Let A be a propositional function containing x as a free variable, and let us for simplicity assume that x is the only free variable in A. By a substitution instance of A is meant the result of replacing 'x' in 'A' by a value of x.

(∃x)A is to be interpreted simply as:

(1) Some substitution instance of A is true.

(x)A is to be interpreted as:

(2) Every substitution instance of A is true.

Alternatively:

(1a) There is at least one value of x for which A is true.

and:

(2a) A is true for every value of x.

On this interpretation, quantification has primarily to do with open sentences, and with truth or falsity. As I will indicate later, much of the misunderstanding about quantification stems from the absence of a standard, unequivocal, colloquial reading of the operations of quantification. This is not to say that they are formalisms completely separate from ordinary speech. (1) corresponds roughly to the non-temporal 'sometimes true' of common speech and (2) to the non-temporal 'always true'. Alternative readings are 'in some cases' and 'in all cases' respectively. Here too the reading is not free of ambiguity, but even in ordinary discourse we have ways of separating out the function peculiar to quantification.

In terms of (1) and (2), one sees clearly the analogues between existential quantification and disjunction on the one hand and universal quantification and conjunction on the other: analogues which are obscured by readings of quantification in terms of 'existence'.

If ontological commitments have to do with existence, no ontological commitments are involved here. Epistemological commitments perhaps, since (1) and (2) involve the notion of truth. Nor is there any justification (except metaphorical) in the contention that the formation rules in terms of which well-formed quantified expressions are constructed *commit* us ontologically. Quantification is tied to the notion of an open sentence and only incidentally to a particular choice of variables. It has to do with the sorting of propositional functions into those which are true in some substitution instances (at least one), and those which are true in all substitution instances. Quantification *need* not be bound to the subject-predicate form unless we choose it as the basic form of a sentence. If we choose as values of the variables the names of things, the names of classes, or the names of properties, then it is no metaphysical mystery that instantiation and quantification will be about things, properties, and classes. The *notion* of quantification, the process involved, like the operations of the propositional calculus, goes beyond the particular choice of basic sentence form. Even as it stands, in an n-order functional calculus ($n \geq 2$), there are instances of quantification over variables which are not individual, class or predicate variables; e.g. '$(\exists p)p$' is well formed where p is a propositional variable. Read in accordance with (1), it says that there is at least one true proposition.

Lacking a straightforward colloquial translation of '(∃x)', the tendency is to standardize it employing such phrases as 'There is' or 'There exists'. I intend to show, by several examples, that such a choice is unfortunate, since it focuses on an accidental feature of quantification rather than an essential one.

In Strawson's account of quantification, he says[1] "the existential quantifier is to be read:

(3) There is (exists) at least one thing (person) which (who)..."

He does not admit the possibility of significant alternative interpretations for he says[2] with reference to (3): "And we might think it strange that the whole of modern formal logic, after it leaves the propositional logic and before it crosses the boundary into the analysis of mathematical concepts, should be confined to the elaboration of sets of rules giving the logical interrelations of formulae which, however complex, all begin with these few rather strained and awkward phrases."

Strawson is justified in pointing out that (3) generates difficulties, as do his analogous readings of expressions involving quantification and negation; e.g. 'There is (exists) nothing which (who)...' One such difficulty (not specifically raised by Strawson) has to do with the theorem of the lower functional calculus

(4) (∃x)A.

Although Russell and Whitehead, in the Introduction to *Principia Mathematica*,[3] substantially laid the groundwork for an interpretation of quantification in terms of (1) and (2), it was subsequently obscured by a reading of (4) in terms of (3) and concluding therefrom that (4) committed us logically to the existence of individuals, and in calculi of higher order to attributes as well. Yet a cursory consideration of any proof of (4) reveals how misleading is such a realistic conclusion. For a proof of (4) consists in generalizing on some tautological function such as 'x=x' or 'Fx v-Fx',or on some tautology such as 'a=a'. If the basic sentence form involves individual variables, such a variable will of course appear in the quantifier. But the import of (4) is that there is at least one true statement involving such variables; and it is true by virtue of being tautologically true.

Turning now to an example raised by Strawson,[4] he says: "...we might try writing the sentence

(5) 'There was at least one woman among the survivors'

in the form

(6) '(∃x) (x is a woman . x was among the survivors)'.

But to say 'There *is* at least one person who is a woman and was among the survivors' is at least to suggest that such a person is alive at the time the sentence is uttered: and no such suggestion is carried by the original sentence. Changing the second 'is' to 'was' will not help; it will merely prompt the question: 'What became of her then? Has she changed her sex?'" Strawson's reading is of course in accordance with (3). Read according to (1) no such perplexities arise. (5) may indeed be written as (6) but interpreted as

(7) Some substitution instance of 'x is a woman and x was among the survivors' is true, where the 'is' of the first conjunct signifies the relationship of attribution.

Admittedly, there is no uniform colloquial counterpart of (7), but Strawson's proposed reading is not based on its apparent advantages for spoken English. He acknowledges it to be "strained and awkward". Indeed the most colloquial equivalent of (6) is (5) itself, for it is least likely to generate spurious questions of tense, being so phrased as to avoid commitment about the present location, spatial or temporal, of the surviving woman.

The absence of a *uniform* colloquial counterpart of (1) does not mean that (1) is not expressible in common speech. There are many devices of locution which are employed for the purpose of avoiding the misunderstandings which follow from Strawson's choice of reading. We might say 'It is sometimes the case that an instance of cancer will show a spontaneous remission' rather than 'There are cases of spontaneous remission in cancer' so as not to be interpreted as restricting the cases to the present. Of the many uses of the verb 'to be', quantification separates out a tenseless use, and, contrary to Strawson's claim, there *are* tenseless uses of 'is' even in contexts involving spatio-temporal objects.

Strawson recognizes that there are tenseless uses of 'there is', as in 'There is at least one prime number between 16 and 20', but he argues that tenseless uses are excluded from contexts involving reference to temporal objects. When confronted with statements such as 'No one loves without somebody suffering', where the present tense is clearly not intended to confine the assertion to the particular moment of utterance, he chooses to interpret such uses as temporal, but omni-temporal. In such statements he says that the 'there is' of quantification must be read as 'there is, or was, or will be'. But what are we to do with 'There are no Catholic American Presidents prior to Kennedy'? And what of the use of the present tense in narrative and historical discourse? Emerson[5] in writing of Plato says: "Nobody can refuse to talk to him, he is so honest, and really curious to know ..." Is this omni-temporal use? Shall we read it 'There is not, was not, or will not be a person who can refuse to talk to him...' and must Plato have been contemporary with Emerson, for Emerson to describe him as honest, in the present tense? Is it not simpler to equate the first part of Emerson's assertion as equivalent to the assertion that no substitution instance of 'x refuses to talk to Plato' is true?

We are not suggesting that when we speak of persons and incidents, questions of time reference do not arise, nor the Parmenidean extension that such questions are illusory. We are suggesting that the operation of quantification is more fruitfully interpreted as independent of tense considerations. However, since spatio-temporal locations can be named as well as persons and things, naturally quantification will enter into any analysis of tense.

Another, and perhaps more significant example of the difficulties generated by (3) is in connection with existential generalization on terms such as 'Pegasus'. Suppose we wish to claim the truth of

(8) Pegasus is a winged horse

and therefore that of

(9) $(\exists x)$ (x is a winged horse).

Reading (9) in accordance with (3) we have the seemingly absurd

(10) There is at least one thing which is a winged horse.

Read in accordance with (1) we have only

(11) There is a true substitution instance of 'x is a winged horse'

and surely *if* we can claim (8) we can claim (11). The problem here is not with quantification so interpreted, but rather the sense in which we can claim (8). The familiar functional calculus, with its paradoxes of material implication and its principles of extensionality is simply too poor a formalism to permit us to claim truth for 'Pegasus is a winged horse'. In such a formalism constants like "Pegasus" may be banned, and only such individual constants which name entities in the "actual" world are tolerated; or the more radical procedure of eliminating singular constants altogether; or either of these in conjunction with a theory of descriptions; or restrictions on existential generalization. Other solutions lying outside the familiar formalism are the denial that (8) has a truth value, or the enrichment of the logic in the direction of intensions, where the truth of (8) may be interpreted relative to some model (true in a possible world). And is the latter not consistent with ordinary use, for *if* one were to claim (8), as we often do, one would be doing so relative to mythology rather than zoology. However, my point here is not to defend intensional logic, but to indicate that just as the notion of quantification is not tied to tense, it also *need* not be tied to a logic which, by making the class of true statements coextensive with the class of true statements about the actual world, precludes the possibility of claiming (8) at all. The difficulty is with making some sense of (8) within the logical framework, not with quantification interpreted as (1) and (2).

The next example of the difficulties which follow from reading the existential quantifier in accordance with (3) occurs in modal logic. It is a theorem of many modal functional calculi that the operator for logical possibility commutes with the existential quantifier. In particular

(12) $\Diamond (\exists x)\, \Phi x \rightarrow (\exists x) \Diamond \Phi x.$

Prior[6] reads (12) as 'If it is possible that something Φ's then there is something which possibly Φ's'. Such a reading is clearly in accordance with (3) and the perplexities generated by such a choice of reading are apparent. The antecedent is about possible objects, but the consequent is about what exists. Reading (12) in accordance with (1) we have

(13) 'If it is possible that there is a true substitution instance of Φx, then it is true that a substitution instance of Φx is possible';

or if $\Diamond$ is taken as logical possibility, it may be read as

(14) 'If it is logically possible that a substitution instance of Φx is true, then it is true that a substitution instance of Φx is logically possible'.

We do not deny that (13) and (14) are clumsy. But they are not paradoxical. We are not making that mysterious move from possibility to actuality. Indeed the sense of (12) can be seen in connection with the *failure* of commutivity between the universal quantifier and the operator for possibility. In such modal systems

(15) $(x) \Diamond A \rightarrow \Diamond (x) A$

is not provable.

Reading (15) in accordance with (3) is very awkward indeed. Reading it in accordance with (2) we have

(16) If it is true that every substitution instance of A is possible, then it is possible that every substitution instance of A is true.

The failure of (15) is analogous to a distinction between the uses of 'each' and 'every'. If $a_1, a_2, \ldots a_n$ are names of presidential candidates, then although every substitution instance of '$\Diamond$ (x is elected president)' is true, '$\Diamond$ (a_1 is elected, a_2 is elected, ... a_n is elected)' is clearly false.

The more familiar example from modal logic has to do with existential generalization in contexts involving modal operators. This difficulty, which is a principal source of Quine's antipathy to modal logic, has again largely to do with the reading of quantification as in (3). It is regarded as paradoxical that from

(17) $\Box$ (Evening Star = Evening Star) (where '$\Box$' is the operator for necessity)

which is regarded as true, it follows by existential generalization that

(18) $(\exists x) \Box (x = \text{Evening Star})$

which is regarded as false.

In the words of (3), (18) becomes 'There is an object...', and we are caught up in the familiar questions. Which object? The Evening Star, which is the same as the Morning Star? I have discussed problems of substitution in modal contexts in some detail elsewhere.[7] Suffice it to say here, that read in accordance with (1), *if* (17) is true, (18) is also true. For surely, given (17), some substitution instance of $\Box$ (x = Evening Star) is true. In particular, that instance where 'x' is replaced by 'Evening Star'.

Several examples have been cited to show that a particular choice of reading may obscure the whole function of the operation of quantification. Yet the choice of (3) is common. This is because in ordinary speech we do not have an unequivocal set of expressions which can be used for the operation of quantification independent of tense, membership, inclusion, actual existence, etc. Where necessary, certain locutions are chosen for the purpose. This is not a failure of ordinary speech, for a source of the power and economy of ordinary language is the plurality of functions which can be assumed by a single expression or group of expressions. One of the tasks of logic is to separate out such uses.

Yet it seems to me that criticism of formal logic such as Strawson's stems from a double-pronged insistence that, on the one hand, all logical expressions have a standard unequivocal reading in ordinary language, as in his choice of reading of quantification, and on the other hand that certain expressions which have a plurality of function in ordinary use, e.g. 'there is', retain the plurality of function in formal analysis. This is a kind of holistic mystique which insists that what has been analyzed cannot be reconstructed.

This is further connected with another unsupportable assumption of ordinary-language criticism of formal logic. Formal logic is identified with the familiar extensional functional calculus, the operations of quantification read as in (3), and the analogies with the class calculus pressed to the extreme. This permits Strawson to say[8] that "the formal logician is reluctant to admit, or even envisage the possibility that his analytical equipment is inadequate for the dissection of most ordinary types of empirical statement".

The rigidity seems to lie rather with Strawson; for many formal logicians freely admit the inadequacy of the familiar functional calculus for the task of analyzing empirical statements. The formal logician's role is analogous to that of the theoretician in any science. If the theory is too disparate with the data his task is to alter the theory, not to abandon theory altogether.

NOTES

[1] P. F. Strawson, *Introduction to Logical Theory*, London, 1952, p. 150.

[2] P. F. Strawson, ibid., p. 147.

[3] A. N. Whitehead and B. Russell, *Principia Mathematica*, Second Edition, Cambridge, 1935, p. xxi.

[4] Op. cit., pp. 150–1.

[5] R. W. Emerson, *Representative Men*, A. L. Burt, New York, p. 66.

[6] A. N. Prior, *Time and Modality*, Oxford, 1957, p. 26.

[7] R. B. Marcus, 'Extensionality', *Mind*, LXIX, N. S., No. 273, pp. 55–62.

[8] P. F. Strawson, Op. cit, p. 216.